Heat Transfer in Film Condensation

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In this theoretical analysis of heat transfer in film condensation the total vapor-side resistance to heat transfer is obtained by adding the resistance to conduction through the condensate film to the resistance due to the condensation process at the vapor-liquid interface.

The analysis shows that under ordinary conditions conduction in the condensate film is controlling, provided the condensation coefficient is greater than about 0.10. However, under conditions of low total pressure, low heat flux, or low condensation coefficient, the resistances of both processes must be considered.

Experiments carried out on the condensation of methanol are in good agreement with the theory. Furthermore, values of the condensation coefficient for methanol are reported which show that small quantities of air will greatly reduce its magnitude. This is offered as an explanation of the commonly observed phenomenon of reduction of heat transfer rates when noncondensable gases enter a system.

Depending upon conditions, a vapor condensing on a solid surface may form either a smooth continuous film or tiny droplets. These two distinct modes of condensation are referred to as *film* and *drop* condensation respectively. Heat transfer in condensation is an extremely complex phenomenon. In general, it depends upon conduction and convection processes in the vapor phase, upon the condensation process at the interface, and upon conduction in the condensate. It is customary in heat transfer calculations to introduce the coefficient h, defined by the equation

$$h = q/A(T_0 - T_2) = q/A\ \Delta T_1 \qquad (1)$$

Nusselt's theoretical analysis of film condensation (*7*), which is discussed in some detail by Jakob (*3*), assumes that heat conduction in the condensate is the rate-controlling step. The Nusselt theory leads to the following equation for the vapor-side coefficient:

$$h_N = Z(Y\ \Delta T_1)^{-1/4} \qquad (2)$$

with Z defined as

$$Z = (g\rho^2\lambda k_L^3/4\mu)^{1/4} \qquad (3)$$

If one assumes that the rate-controlling step is the conduction of heat through the condensate, the temperature drop from the bulk of the vapor to the interface must be negligible. Under most operating conditions this is a very reasonable assumption and many experimenters (*1, 2, 6, 8, 9,* and *11*) have obtained results that agree quite closely with the Nusselt theory.

Under some operating conditions, however, notably when total pressures are low and when small quantities of noncondensable gases are present, the Nusselt theory consistently gives values for h which are larger than those observed. The explanation must be that heat conduction through the condensate is no longer in complete control and that the other processes must be considered. The analysis presented here considers both conduction in the condensate and the condensation process at the interface.

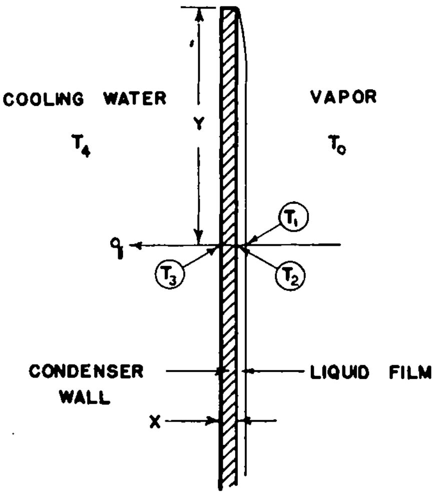

Fig. 1. Filmwise condensation on a vertical surface.

THEORETICAL ANALYSIS

In Figure 1 a pure vapor at temperature T_0 and pressure P_0 is shown condensing in a film on a solid vertical surface. At the distance Y from the top of the surface the interface temperature is T_1, the front surface of the condenser wall is at temperature T_2, and the back surface is at temperature T_3.

An equation for the heat flux through the liquid film is obtained from (1) and (2) by replacing T_0, the vapor bulk temperature, with T_1, the interface temperature.

$$(q/A) = K_2(\Delta T_3)^{3/4} \qquad (4)$$

where

$$K_2 = Z(Y)^{-1/4}$$

$$\Delta T_3 = (T_1 - T_2)$$

At the vapor-liquid interface, condensation and reevaporation occur rapidly, with the condensation rate slightly exceeding the evaporation rate. Evaporating molecules leave the liquid surface with an average temperature T_1. Molecules striking the interface come from a region in the vapor that is very close to the interface. In this region molecules which have just evaporated and are at the average temperature T_1 collide with molecules coming from the bulk of the vapor at an average temperature T_0. An average vapor temperature T^*, which is intermediate between T_0 and T_1, exists in the small region next to the vapor-liquid interface. Molecules striking the interface are at the average temperature T^*.

The rate at which molecules strike the surface is given (*4*) by the kinetic-theory formula

$$r_0 = P_0/(2\pi R'T^*/M)^{1/2} \qquad (5)$$

All molecules which strike the surface do not necessarily condense; consequently it is necessary to introduce the condensation coefficient α, which represents the fraction of molecules striking the surface which do condense. The absolute rate of condensation r_1 is obtained by multiplying r_0 by α.

If the vapor and the liquid surface were at the same temperature, the net rate of condensation would be zero, as the absolute condensation rate would be just equal to the absolute evaporation rate. Consequently, the absolute evaporation rate can be calculated for any given temperature. For a liquid surface at temperature T_1

$$r_2 = P_1/(2\pi R'T_1/M)^{1/2} \qquad (6)$$

The net rate of condensation r is obtained by subtracting the absolute evaporation rate r_2 from the absolute condensation rate r_1.

$$\begin{aligned} r &= (r_1 - r_2) \\ &= [\alpha P_0/(2\pi R'T^*/M)^{1/2}] \qquad (7) \\ &\quad \cdot [1 - (P_1/P_0)(1 + \Delta T^*/T_1)^{1/2}] \end{aligned}$$

where

$$\Delta T^* = (T^* - T_1)$$

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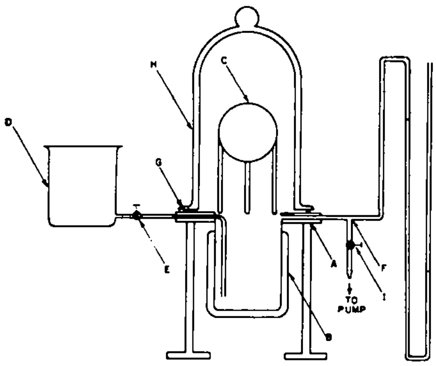

Fig. 2. Schematic diagram of apparatus.

The vapor pressures P_0 and P_1 can be related to temperatures T_0 and T_1 through the Clausius-Clapyron equation.

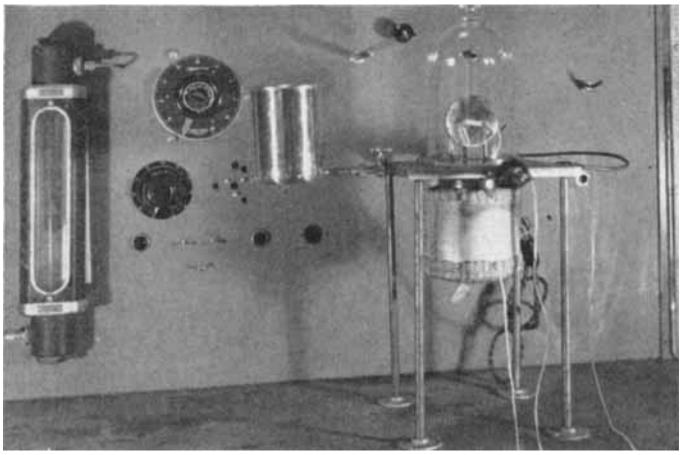

Fig. 3. Photograph of apparatus.

$$(P_1/P_0) = \exp(-\lambda M \ \Delta T_0/RT_1T_0) \quad (8)$$

where

$$\Delta T_0 = (T_0 - T_1)$$

For the case where ΔT_0 and ΔT^* are of the order of a few degrees, Equation (7) can be simplified by the series expansions shown below.

$$\exp(-\lambda M \ \Delta T_0/RT_1T_0) = 1 - (\lambda M \ \Delta T_0)/(RT_1T_0) - \cdots \quad (9)$$

$$(1 + \Delta T^*/T_1) = 1 + \tfrac{1}{2}(\Delta T^*/T_1) + \cdots \quad (10)$$

Combining (7), (8), (9), and (10), and neglecting terms containing ΔT_0 and ΔT^* to powers greater than one gives the net rate of condensation as

$$r = [(\alpha P_0 \Delta T_0/T_1)(2\pi R'T^*/M)^{-1/2}] \ [(\lambda M)/(RT_0) - \tfrac{1}{2}(\Delta T^*/\Delta T_0)] \quad (11)$$

Under most conditions the second term on the right side of (11) can be neglected. For example, for methanol at 50°C. the value of $(\lambda M/RT_0)$ is 13.4. Since the ratio of ΔT^* to ΔT_0 is less than 1, the second term amounts to less than 4% of the first. Similarly, T_1 and T^* in the denominator of the coefficient of the right side of (11) can be replaced with T_0 with very little error. With these simplifications the net rate of condensation becomes

$$r = [(\alpha P_0 \lambda M \ \Delta T_0)/(RT_0^2)] \cdot [(2\pi R'T_0/M)^{-1/2}] \quad (12)$$

The rate of heat evolution at the liquid surface due to condensation is obtained by multiplying the rate of condensation by the latent heat of vaporization. Therefore, the heat flux is given by the equation

$$(q/A) = \alpha K_1 \ \Delta T_0 \quad (13)$$

where

$$K_1 = [(\lambda^2 M P_0)/(RT_0^2)] \cdot [(2\pi R'T_0/M)^{-1/2}]$$

Therefore, from Equations (4) and (13)

$$K_2(\Delta T_3)^{3/4} = \alpha K_1(\Delta T_0) \quad (14)$$

Denoting the temperature difference between the vapor and the condenser surface by ΔT_1 and noting that

$$\Delta T_1 = \Delta T_0 + \Delta T_3 \quad (15)$$

one obtains an equation for the film coefficient h by combining (15), (14), and (1):

$$(1/h) = (q/A)^{1/3}/(K_2)^{4/3} + (1/\alpha K_1) \quad (16)$$

In (16) the total vapor-side resistance is shown to be equal to the sum of two additive resistances, the first due to conduction in the liquid film and the second to the condensation process at the interface. Under certain conditions the second term becomes negligibly small and (16) reduces to the Nusselt equation.

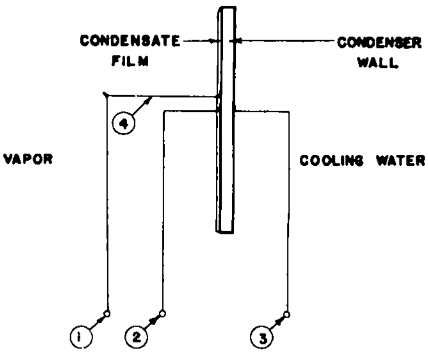

Fig. 4. Diagram of thermocouple circuit.

Since heat is transferred through the condenser plate by conduction, the heat flux is given by the equation

$$(q/A) = K_3 \ \Delta T_2 \quad (17)$$

where

$$K_3 = k/x$$

$$\Delta T_2 = (T_2 - T_3)$$

A convenient equation for testing the theoretical analysis is obtained by combining (17), (16), and (1):

$$(\Delta T_1/\Delta T_2) = (K_3/\alpha K_1) + (K_3/K_2)^{4/3}(\Delta T_2)^{1/3} \quad (18)$$

Since K_1, K_2, and K_3 can be calculated directly from known physical properties, it is possible to determine α from an experiment in which ΔT_1 and ΔT_2 are varied, but the vapor temperature, pressure, and α are constant. In this case a plot of $(\Delta T_1/\Delta T_2)$ vs. $(\Delta T_2)^{1/3}$ on linear graph paper should be a straight line.

EXPERIMENTAL

The apparatus designed for the experimental studies is shown schematically in Figure 2, and an over-all view is given in Figure 3. It consists of a stainless steel plate (*A*) with a circular hole into which a stainless steel beaker, which serves as a boiler for the system, is welded. A glass bell jar (*H*) fits over the beaker, a vacuum-tight seal being formed with an 0 ring (*G*), which fits in a groove in the plate. The condenser plate (*C*) is a pure nickel disk behind which cooling water is circulated. Provision is made to circulate the cooling water at a constant rate and at any temperature from about 25° to about 95°C.

The thermocouple circuit consists of four wires and the nickel disk. It is shown schematically in Figure 4. Wires 1, 2, and 3 are 30-gauge Manganin, and wire 4 is 30-gauge constantan. Wires 2, 3, and 4 were attached by being peened into shallow grooves in the surface of the nickel disk. A Leeds and Northrup model K-2 potentiometer was used to make the electromotive-

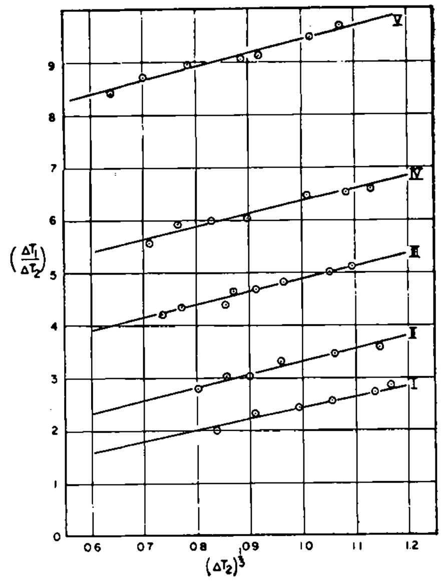

Fig. 5. Correlation of data.

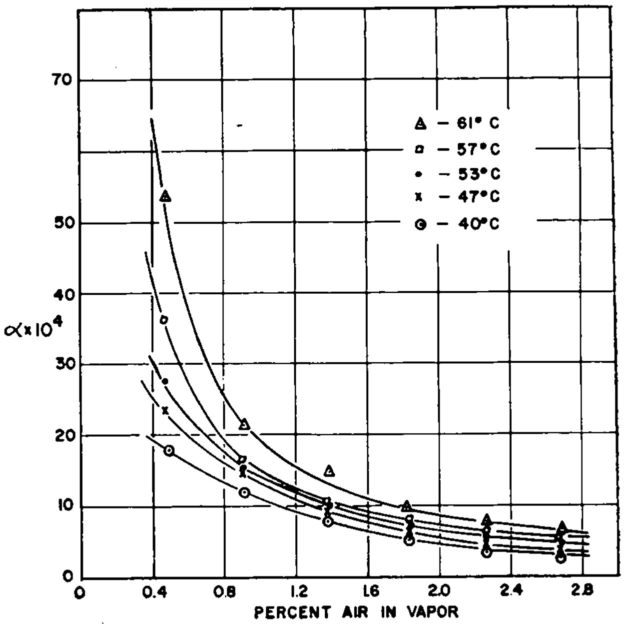

Fig. 7. Effect of air concentration on the condensation coefficient.

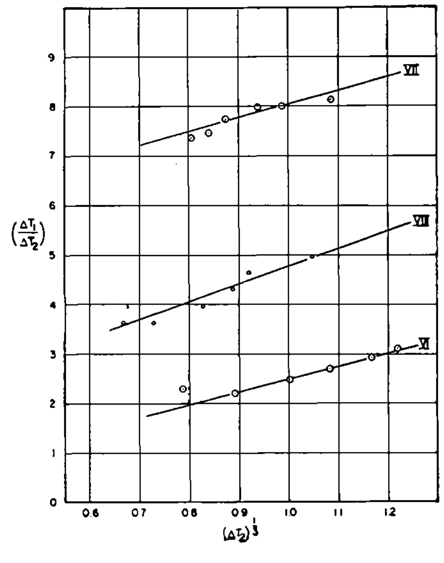

Fig. 6. Correlation of data.

force measurements. With wires 1 and 2 connected to the potentiometer, the net electromotive force is proportional to ΔT_1. When wires 2 and 3 are connected to the potentiometer, the net electromotive force is proportional to ΔT_2.

Since the thermocouple wires are of small diameter and of relatively low thermal conductivity, and temperature gradients in the system are small, errors due to heat conduction in the wires are very small. Under the operating conditions encountered in the experiments the maximum error in the thermocouple readings due to heat conduction is estimated to be about 0.01°C.

To fill the boiler, valve E is closed and the system evacuated. Valve I is then closed, liquid placed in beaker D, and valve E opened. The liquid is then allowed to flow into the boiler until the liquid level rises almost to the top. Valve E is then closed and the temperature of the liquid brought up to the desired level by regulating the voltage applied to the "Glas-Col" heating mantle (B). The pressure in the system is indicated by the mercury manometer, which is connected to the system through the line F.

As the vapor condenses on the surface of the nickel disk, it forms a film, runs down the surface of the disk, and drops back into the boiler. During a run the bell jar is jacketed with a glass-wool blanket to reduce heat losses. Measurements of ΔT_1 and ΔT_2 are made until a steady state is reached. By changing the temperature of the cooling water, ΔT_1 and ΔT_2 can be varied. The vapor pressure and temperature can be maintained constant by adjusting the heat input from the heating mantle.

DATA AND RESULTS

Methanol was selected for the experimental work because its physical properties are well known and it has a convenient vapor pressure in the working temperature range of the apparatus. Figure 5 is a plot of the data obtained from the first five runs. During each run the vapor temperature and pressure were maintained constant. As predicted by Equation (18), the data for each run fall on a straight line. The slope and intercept are given by the following formulas derived from (18):

$$\text{slope} = (K_3/K_2)^{4/3} = (kY^{1/4}/xZ)^{4/3} \qquad (19)$$

$$\text{intercept} = (K_3/\alpha K_1) \cdot [(kRT_0^2)/(\alpha x \lambda^2 M P_0)] \cdot [(2\pi R'T/M)^{1/2}] \qquad (20)$$

The results of these runs are summarized in Table 1, where the theoretical values of the slopes, as calculated with (19), are compared with the experimentally obtained values. The deviation between theory and experiment is about ±8%, which is about the estimated experimental error. The condensation coefficient α is calculated from (20), by use of the measured value of the intercept. Values of α for these runs are also shown in Table 1.

The small magnitude and the large, but apparently systematic, variation of α in runs I through V was unexpected. It was considered unlikely that this variation in α could be caused by the relatively small change in temperature or total pressure. It was considered probable that very small quantities of noncondensable gases could have a large effect on α. In runs I through V there was undoubtedly a small residual amount of air left in the system after evacuation. This residual air remained in the system during the runs. Since the system was sealed, the total quantity of air was constant, but the concentration of air, being inversely proportional to total pressure, changed from run to run.

Runs VI, VII, and VIII were designed to investigate the effect of air on the condensation coefficient. In run VI the system was evacuated, the methanol was introduced and then the methanol vapor, together with any residual air, was quickly withdrawn through the pump. The pump was then shut off, the system allowed to return to equilibrium, and the vapor again withdrawn. This procedure was repeated twenty times, so that most of the residual air was probably eliminated. Upon completion of this run, air was deliberately introduced into the system and the run repeated under the same conditions of temperature and pressure. This is run VII in Table 1. Run VIII was carried out in the same way as VI (that is, the system was purged of residual air) except that ethyl ether was substituted for methanol.

The data for these three runs are shown in Figure 6. Again, the straight-line relationship was obtained and the theoretical and experimental slopes were in reasonable agreement. It is interesting to note, however, that with methanol the condensation coefficient was decreased by a factor of about 20 when 1% air was introduced. These two runs, therefore, show that small quantities of air can have a drastic effect on α.

The purpose of run VIII, in which ether was substituted for methanol, was to see whether the polarity of the condensing substance would affect the magnitude of the condensation coefficient. Runs VI and VIII, made under essentially the same operating conditions, indicate that polarity is probably not an important factor.

The final series of runs was a systematic attempt to determine the effect of air on α for methanol. Runs were made at five different vapor temperatures and at six different air concentrations. The range of temperatures covered was from about 40° to 61°C., and the air concentration was varied from about 0.4 to 2.8%. The value of α was determined for each run and the results are shown in Figure 7.

DISCUSSION OF RESULTS

The experimental results show that small quantities of air will greatly reduce the value of α for methanol and that this, in turn, reduces the heat transfer coefficient. Direct comparison of the values of α obtained in this work with those of previous investigators is not possible, as, according to Schrage (*10*), other investigations have been limited to relatively nonvolatile substances or to very low temperatures; however, there is some evidence that values of α can vary over wide limits. Knudsen (*5*), for example, found that condensation coefficients for mercury varied between 0.0005 and unity, depending upon the purity of the mercury.

In an analysis of the experimental data it is instructive to consider the possibility that α does not depend upon air concentration, but that its apparent variation is due to diffusion effects. The diffusion process to be considered is the steady state diffusion of one gas (methanol vapor) through a second stagnant gas (air). Approaching the interface from the bulk of the vapor, the air concentration increases and the methanol vapor concentration decreases. The result of diffusion, therefore, is a lower partial pressure of vapor next to the interface and, consequently, a reduced condensation rate.

Under these conditions the condensation rate can be calculated with an equation similar in form to (7) if the total pressure P_0 is replaced with the partial pressure p of vapor next to the interface.

$$r = [(\alpha p)/(2\pi R'T^*/M)^{1/2}] \cdot [1 - (P_1/p)(1 - \Delta T^*/T_1)^{1/2}] \qquad (21)$$

p depends upon the diffusivity of methanol vapor in air, the bulk concentration of air, the condensation rate, the thickness of the concentration boundary layer, and the vapor temperature and pressure. The factor γ, defined by the equation

$$\gamma = (p/P_0) \qquad (22)$$

is introduced into (21) and, after the same simplifying assumptions as before are made, the following result is obtained:

$$(q/A) = \alpha\gamma K_1\, \Delta T_0 - K_0 \qquad (23)$$

where

$$K_0 = [(1 - \gamma)\alpha\lambda P_0]/(2\pi R'T_0/M)^{1/2}$$

For a pure vapor γ is equal to unity and (23) reduces to (13). If (23) is combined with (17), (16), and (3), as before

$$(\Delta T_1/\Delta T_2) = [(K_3/\alpha K_1) + (K_0/\alpha\gamma K_1)(1/\Delta T_2)] + (K_3/K_2)^{4/3}(\Delta T_2)^{1/3} \qquad (24)$$

Equation (24) shows that if diffusion were of any consequence the linear relationship between $(\Delta T_1/\Delta T_2)$ and $(\Delta T_2)^{1/3}$ would no longer be valid. The data therefore force one to the conclusion that diffusion effects were negligible in the present work.

In the apparatus used for the experimental work the vapor was essentially at rest. In ordinary condensers the vapor velocities are much higher, and it seems probable that diffusion effects would be of no importance, provided, of course, that the noncondensable gas concentration is of the order of a few per cent. The deleterious effect of small quantities of air must, therefore, be due to the adverse effect of air on the condensation coefficient. This conclusion differs from those reached by some previous investigators. Othmer (*8*), for example, in his analysis of heat transfer in condensation came to the conclusion, on the basis of a rate equation somewhat similar to (7), that the resistance to condensation at the interface was negligible. However, he assumed, implicitly, in his analysis that the condensation coefficient was unity. The present work shows that this assumption is not valid.

Conventionally, the vapor-side heat transfer coefficient for film condensation would be calculated with the Nusselt equation, which can be written

$$(1/h_N) = (q/A)^{1/3}/(K_2)^{4/3} \qquad (25)$$

The work presented in this paper shows that a better approximation to the total vapor-side resistance is given by (16).

$$(1/h) = (q/A)^{1/3}/(K_2)^{4/3} + (1/\alpha K_1) \qquad (16)$$

If ϵ is defined as (h/h_N), then

$$\epsilon = (1)/[1 + (h_N)/(\alpha K_1)] \qquad (26)$$

where

$$(h_N)/(\alpha K_1) = \left[\frac{RT_0^2(2\pi R'T_0/M)^{1/2}}{\alpha(q/A)^{1/3}(YZ)^{1/3}\lambda^2 MP_0}\right]$$

ϵ can vary between zero and unity. For values of ϵ approaching unity, heat conduction in the condensate film is controlling and the Nusselt theory gives a good approximation to the vapor-side resistance. On the other hand, Equation (26) shows that under certain conditions ϵ can be very small, an indication that

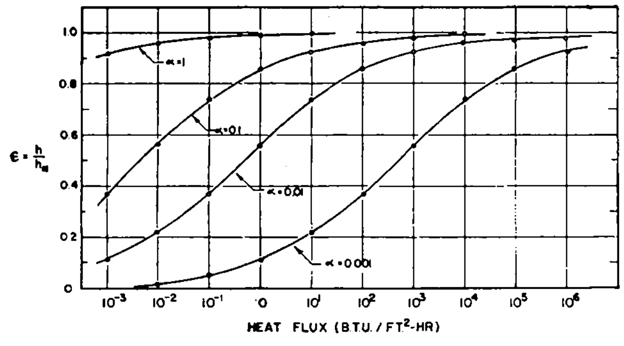

Fig. 8. Effect of the condensation coefficient on heat transfer rates.

the rate of the condensation process at the interface must be considered when the total vapor-side resistance is calculated. This condition will arise at low heat flux, low values of α, and low total pressures.

Figure 8 shows, as an example, how the condensation coefficient α can affect ϵ. The figure applies to saturated methanol vapor at 1 atm. and shows that if α is near unity, then regardless of the heat flux, ϵ will also have values near unity. If α should have a value of 10^{-3}, on the other hand, then even at a heat flux as high as 10^3 B.t.u./(hr./sq. ft./°F.) ϵ will be reduced to almost 0.5.

The over-all rate of heat transfer through a plane condenser wall can be expressed in terms of an over-all coefficient, defined by the equation

$$(1/U) = (1/h_w) + (x/k) + (1/\epsilon h_N) \tag{27}$$

TABLE 1.—SUMMARY OF RESULTS

Run	Pressure, mm. Hg	Temperature, °K.	Slope Theory	Experiment	$\alpha \times 10^4$	Vapor	Comments
I	620	333	2.29	2.1	56.0	Methanol	In runs I through V a small but constant quantity of air was present in the system.
II	552	330	2.29	2.5	27.1	Methanol	
III	484	327	2.33	2.3	10.2	Methanol	
IV	374	321	2.35	2.3	7.9	Methanol	
V	281	314	2.40	2.6	5.7	Methanol	
VI	209	308	2.44	2.6	164.0	Methanol	System purged
VII	215	309	2.44	2.7	8.6	Methanol	Air introduced
VIII	683	305	4.44	4.5	184.0	Ether	System purged

If it is assumed that changes in the vapor-side resistance have no effect on the other two resistances, then the change in U due to a change in ϵ is given by the formula

$$(U_2/U_1) = (\epsilon_2/\epsilon_1) \cdot [(\epsilon_1\beta + 1)/(\epsilon_2\beta + 1)] \tag{28}$$

where

$$\beta = h_N[(1/h_w) + (x/k)]$$

As an example saturated methanol vapor may be considered at 1 atm. condensing in a film at a heat flux of 10^3 B.t.u./(hr./sq. ft./°F.) in an air-free system where the vapor-side resistance is about 20% of the total resistance to heat transfer. The reduction in the over-all coefficient to be expected if 2% air enters the system is shown by the following calculation.

From Figure 7 the initial value of α is estimated to be greater than 0.1 and from Figure 8 the value of ϵ is taken to be about 0.98. With 2% air, α would be reduced to about 10^{-3} and ϵ becomes about 0.55. For the initial condition, h_N can be taken as equal to h. From its definition,

$$\beta = (0.80)/(0.20) = 4$$

and

$$(U_2/U_1) = (0.55/0.98)[(0.98/4 + 1)/(0.55/4 + 1)] = 0.86$$

The introduction of 2% air causes a 14% reduction in the over-all coefficient. If the condenser were operating at a higher heat flux, the effect would be less severe, and at lower heat flux it would be much more severe.

ACKNOWLEDGMENT

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NOTATION

α = condensation coefficient, dimensionless
β = dimensionless group defined by (28)
γ = dimensionless ratio defined by (22)
ϵ = dimensionless group defined by (26)
λ = latent heat of vaporization, cal./gram
μ = viscosity of condensate, poise
ρ = density of condensate, grams/cc.
g = gravitational acceleration, 980 cm./sec.
h = vapor-side coefficient, cal./(sec./sq. cm./°C.)
h_w = water-side coefficient, cal./(sec./sq. cm./°C.)
h_N = vapor-side coefficient according to Nusselt theory, cal./(sec./sq. cm./°C.)
k = thermal conductivity of condenser wall, cal./(sec./cm./°C.)
k_L = thermal conductivity of condensate, cal./(sec./cm./°C.)
p = partial pressure of vapor next to interface, dynes/sq. cm.
(q/A) = heat flux, cal./(sq. cm./sec.)
r_0 = rate at which vapor strikes interface, grams/(sq. cm./sec.)
r_1 = absolute rate of condensation, grams/(sq. cm./sec.)
r_2 = absolute rate of vaporization, grams/(sq. cm./sec.)
r = net rate of condensation, grams/sq. cm./sec.
x = thickness of condenser wall, cm.
K_0 = coefficient defined by (23)
K_1 = coefficient defined by (13)
K_2 = coefficient defined by (4)
K_3 = coefficient defined by (17)
M = molecular weight of vapor
P_0 = vapor pressure at T_0, dynes/sq. cm.
P_1 = vapor pressure at T_1, dynes/sq. cm.
R = gas constant, 1.98 cal./(mole/°C.)
R' = gas constant, 8.31×10^7 ergs/(mole/°C.)
T_0 = bulk vapor temperature, °K.
T_1 = vapor-liquid interface temperature, °K.
T_2 = temperature of front surface of condenser wall, °K.
T_3 = temperature of back surface of condenser wall, °K.
T_4 = bulk temperature of cooling water, °K.
T^* = average vapor temperature in region of interface
U = over-all coefficient for heat transfer, cal./(sq. cm./sec./°C.)
Y = distance from top of condenser wall, cm.
Z = function defined by (3)
ΔT_0 = $(T_0 - T_1)$
ΔT_1 = $(T_0 - T_2)$
ΔT_2 = $(T_2 - T_3)$
ΔT_3 = $(T_1 - T_2)$
ΔT^* = $(T^* - T_1)$

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